Prepared for submission to JCAP

Gradient expansion of superhorizon perturbations in Galileon inflation

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Abstract. We develop the gradient expansion formalism for shift-symmetric Galileon-type actions. We focus on backgrounds that undergo Galileon inflation, work in the synchronous gauge, and obtain a general solution up to second order without imposing extra conditions at first order. The solution simplifies during the late stages of Galileon inflation. We also define a curvature perturbation conserved up to first order.

Keywords: Modified Gravity, Cosmological Perturbation Theory, Inflation, Non-Gaussianity.

Contents

1 Introduction

Inflation is a powerful paradigm that addresses various fine-tuning problems in the early Universe and accounts for the nearly scale invariant primordial perturbations that are needed for structure formation. These primordial perturbations leave an imprint on the cosmic microwave background (CMB). Single field slow-roll inflation models (with a canonical kinetic term) generically predict a Gaussian spectrum of primordial perturbations [1]. However, despite the success of the inflationary paradigm, its theoretical underpinning is still a matter of debate. Hence, it is not clear why one should remain within the framework of slow-roll inflation or even single-field inflation. Substantial non-Gaussianity can be generated in inflation models with multiple scalar fields or with non-canonical kinetic terms. Furthermore, if the slow-roll condition is temporarily violated, large non-Gaussianity can be generated even in a single field model [2]. On the observational side, results from WMAP are consistent with a Gaussian spectrum of primordial perturbations [3] and the recently released results from PLANCK are already leading to tighter constraints of non-Gaussianity [4]. With the prospect of probing the inflation scenario much deeper, non-Gaussianity in different inflation models has been extensively investigated and classified during the last decade (for a review, see [5, 6]).

To tackle non-Gaussianity from inflation models, traditional linear perturbation theory is inadequate. A natural approach is to go beyond the linear order and work with second order cosmological perturbation theory [1, 7–12]. While this approach usually applies to primordial perturbations up to the horizon exit, an alternative approach naturally tackles the superhorizon perturbations – gradient expansion [13–25]. In gradient expansion, physical

quantities are expanded in terms of their inverse wavelengths, as compared to a pivotal length scale ($\epsilon \sim L_p/L_{phys}$), so every spatial derivative adds one perturbative order, $\partial_i \sim \epsilon$, hence th e name. This is different from usual cosmological perturbation theory where the expansion is in terms of perturbative field amplitudes. In the context of cosmology, particularly in the inflationary epoch when physical modes are stretched well outside the quasi-constant Hubble horizon, the Hubble length can be naturally chosen as the pivotal length scale. Therefore, this approach can be used to evaluate and evolve non-Gaussiantites at superhorizon scales, complementary to usual non-linear perturbation theory. The leading order gradient expansion is often called the separate universe approach [26] or δN formalism [27, 28], which is sufficient for many purposes. However, the next-to-leading order gradient corrections can be as important, for example, in some multi-field models or when the slow-roll condition is violated [25, 29]. A beyond-δN formalism scheme has recently been proposed [25].

The Galileon is a scalar field with a galilean(-like) symmetry $\phi \to \phi + b_\mu x^\mu + c$ (b_μ, c being constant) around flat space, and was originally introduced as an effective, infrared gravitational modification which can lead to self-accellerating solutions [30]. The Galileon Lagrangian contains higher order derivatives but nevertheless leads to second order equations of motion, thus avoiding Ostrogranski ghosts.

Coupling the Galileon covariantly to gravity and insisting on the requirement that the scalar and the metric satisfy second order equations forces one has to abandon galilean symmetry [31]. Ordinary shift symmetry $\phi \to \phi + c$ can be retained [32] or abandoned as well. In the latter case, covariant actions for generalized Galileons coupled to gravity have been constructed in Refs. [33, 34]. The 4-dimensional version of the action given in Ref. [34] has been shown [35] to be equivalent to the most general action for a scalar coupled to gravity that leads to second order equations of motion, given by Horndeski in the 1970s [36]. The Galileon model has also been generalized in various other directions (see e.g. [37–43] and references therein) .

The self-accelerating solutions of Galileon models have been the basis of inflationary scenarios [32, 35, 44–50]. There are some known novel features in Galileon inflation (or G-inflation [32]): The null energy condition can be drastically violated without developing instabilities [44]; A large tensor-to-scalar power spectrum ratio is allowed [32]; There are new shapes of the three-point function and potentially large four-point function [45].

In this paper, we develop the superhorizon gradient expansion formalism for Galileon inflation, up to second order in gradient expansion. We focus the subclass of actions for which the scalar enjoys shift symmetry, as they are closer to the original idea of the Galileon and significantly simpler. Additionally, our goal is to explore the phenomenology associated with the non-linear derivative interactions of the scalar. Abandoning shift symmetry leads, amongst other terms, to allowing a potential for the scalar, which can lead to similar phenomenology and, therefore, obscures the role of the Galileon-type terms. We derive the general solution for an Friedman–Lemaître–Robertson–Walker (FLRW) background, identify the degrees of freedom in the model and define a curvature perturbation conserved up to $\mathcal{O}(\epsilon)$. We also consider how our results simplify in the limit where the background becomes de Sitter spacetime. Given that the latter is an attractor of G alileon inflation, this approximation provides a good description at least for the later stages of inflationary expansion.

During the preparation of this manuscript ref. [24] appeared, which also develops the superhorizon gradient expansion formalism for G-inflation without assuming shift symmetry. However, there are major differences: On the technical side, we work in the synchronous

gauge, while ref. [24] prefers the uniform expansion gauge; On the more substantial side in ref. [24] it is assumed that $\partial_t h_{ij}(t,x)$ (see eq. 4.3) is $\mathcal{O}(\epsilon^2)$, while we do not impose such condition. In this respect our results are more general.

The paper is organized as follows. In Section 2, we specify the model to investigate and write down the equations of motion and in Section 3 we perform $3+1$ decomposition of the equations of motion. In Section 4, we establish the gradient expansion orders of relevant quantities. The equations of motion are solved up to $\mathcal{O}(\epsilon^2)$ in Section 5, and the general solution is summarized in Section 5.4. In Section 6, we simplify the general solution in the de Sitter limit. Note that in Section 5, Section 6 and Appendix B, we mostly suppress the background quantities' order indication $^{(0)}$ to simplify the equations. Section 7 contains our conclusions.

2 Inflation with a shift-symmetric Galileon

Following the arguments in the Introduction, we consider the action

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_{pl}^2}{2}R + K(X) - G(X)\Box\phi\right), \tag{2.1}$$

where M_{pl} is the Planck mass, R is the Ricci scalar and $K(X)$ and $G(X)$ are general functions of $X = -\partial_\mu\phi\partial^\mu\phi/2$. Note that for $G = 0$ we recover k-inflation [51], and the model with $K = X, G = \alpha X$ (α =const.) is the cubic covariant Galileon [31], which in turn is linked to the DGP model [52]. The equations of motion for the metric, to which we will refer as Einstein equations, are given by

$$M_{pl}^2 G_{\mu\nu} = T^\phi_{\mu\nu}, \tag{2.2}$$

with

$$T^\phi_{\mu\nu} = (K_X - G_X\Box\phi)\partial_\mu\phi\partial_\nu\phi - 2\partial_{(\mu}G\partial_{\nu)}\phi + g_{\mu\nu}(K + \partial_\sigma G\partial^\sigma\phi), \tag{2.3}$$

where a subscript X denotes partial differentiation with respect to X. Note that the energy momentum tensor takes the form of an imperfect fluid, thus this model does not fall under the existing formalism for a perfect fluid [20]. Thanks to the shift symmetry, the equation of motion for ϕ can be given in terms of the current

$$J^\mu = (K_X - G_X\Box\phi)\partial^\mu\phi - G_X\partial^\mu X, \tag{2.4}$$

as

$$\nabla_\mu J^\mu = 0. \tag{2.5}$$

It is worth mentioning that the scalar equation of motion is implied by the Einstein equations, i.e., once the Einstein equations are satisfied, the scalar equation of motion is automatically satisfied. In fact, this applies to any covariant scalar-tensor system, as we show in Appendix A.

3 $3+1$ decomposition

Now, we perform $3+1$ decomposition of the equations of motion. First, we decompose the metric according to the Arnowitt-Deser-Misner (ADM) prescription

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + \gamma_{ij}(\mathrm{d}x^i + N^i\mathrm{d}t)(\mathrm{d}x^j + N^j\mathrm{d}t). \tag{3.1}$$

To reduce redundant gauge degrees of freedom and simplify equations, we make use of a gauge condition:

$$N = 1, \qquad N^i = 0, \tag{3.2}$$

which implies

$$g_{tt} = -1, \quad g_{ti} = 0, \quad g_{ij} = \gamma_{ij}, \tag{3.3}$$
$$g^{tt} = -1, \quad g^{ti} = 0, \quad g^{ij} = \gamma^{ij}. \tag{3.4}$$

Here latin indices (except for t) run from 1 to 3 and they are raised and lowered with γ^{ij} and γ_{ij} respectively. This is called synchronous gauge, where the proper time distance between two neighboring hypersurfaces along the normal vector coincides with the coordinate time distance ($N = 1$, proper time slicing) and the spatial coordinates are such that clocks are synchronized between different hypersurfaces ($N^i = 0$). In synchronous gauge, equations can be very much simplified. Note, however, that there is residual gauge freedom, which will be discussed in section 5.4.

Now, the spatial γ_{ij} can be considered as a fundamental dynamical variable. Another fundamental variable after the $3+1$ decomposition is the extrinsic curvature, which in synchronous gauge is simply

$$\mathcal{K}_{ij} = -\Gamma^t_{ij} = -\frac{1}{2}\dot{\gamma}_{ij}, \tag{3.5}$$

and its trace is defined as $\mathcal{K} = \gamma^{ij}\mathcal{K}_{ij}$. It is useful to decompose the spatial metric and the extrinsic curvature as

$$\gamma_{ij} = a^2(t) e^{2\zeta(t,\mathbf{x})} h_{ij}(t,\mathbf{x}), \tag{3.6}$$
$$\mathcal{K}^i_j = \frac{1}{3}\mathcal{K}(t,\mathbf{x})\delta^i_j + A^i_j(t,\mathbf{x}), \tag{3.7}$$

where $a(t)$ is the scale factor of the fiducial FLRW background, $\zeta(t,\mathbf{x})$ is related to the curvature perturbation, $h_{ij}(t,\mathbf{x})$ is defined to have a unit determinant $\det[h_{ij}] = 1$, and A^i_j is the traceless part of $\mathcal{K}^i_j$. These definitions lead to the following relations

$$\mathcal{K} = -3\left[\frac{\dot{a}}{a} + \dot{\zeta}\right], \tag{3.8}$$
$$\dot{h}_{ij} = -2h_{ik}A^k_j. \tag{3.9}$$

To decompose the equations of motion, we first make use of some well-known results which do not make reference to any specific gauge. Using the ADM variables, the unit normal 1-form and vector can be written respectively as $n_\mu = (-N, 0, 0, 0)$ and $n^\mu = (1/N, -N^1/N, -N^2/N, -N^3/N)$. Making use of the Gauss-Codazzi relations (see e.g. [53]), we can write the Ricci tensor and Ricci scalar respectively as

$$\begin{aligned} R_{\mu\nu} = {} & n_\mu n_\nu \left(\frac{1}{N}\mathcal{L}_{\mathbf{m}}\mathcal{K} + \frac{1}{N}\mathrm{D}^\lambda \mathrm{D}_\lambda N - \mathcal{K}^\rho_\sigma \mathcal{K}^\sigma_\rho\right) - 2n_{(\mu}\mathrm{D}_{\nu)}\mathcal{K} + 2n_{(\mu|}\mathrm{D}_\sigma \mathcal{K}^\sigma_{|\nu)} \\ & - \frac{1}{N}\mathcal{L}_{\mathbf{m}}\mathcal{K}_{\mu\nu} - \frac{1}{N}\mathrm{D}_\mu \mathrm{D}_\nu N + {}^{[3]}R_{\mu\nu} + \mathcal{K}\mathcal{K}_{\mu\nu} - 2\mathcal{K}^\sigma_\mu \mathcal{K}_{\nu\sigma}, \end{aligned} \tag{3.10}$$
$$R = {}^{[3]}R + \mathcal{K}^2 + \mathcal{K}^\rho_\sigma \mathcal{K}^\sigma_\rho - \frac{2}{N}\mathcal{L}_{\mathbf{m}}\mathcal{K} - \frac{2}{N}D^\sigma D_\sigma N, \tag{3.11}$$

where $m^\mu = Nn^\mu$, $\mathcal{L}_\mathbf{m}$ is the Lie derivative along m^μ, and D_μ is the covariant derivative, ${}^{[3]}R$ the Ricci scalar, and ${}^{[3]}R_{\mu\nu}$ the Ricci tensor of the spacelike hypersurfaces. The Laplacian is decomposed as

$$\Box\phi = -n^\rho\partial_\rho(n^\sigma\partial_\sigma\phi) + \mathcal{K}n^\sigma\partial_\sigma\phi + \mathrm{D}^\sigma \ln N\partial_\sigma\phi + \mathrm{D}^\sigma\mathrm{D}_\sigma\phi. \tag{3.12}$$

In the synchronous gauge ($N = 1, N^i = 0$), the Einstein equations are greatly simplified:

$$M_{pl}^2 G_{tt} = T^\phi_{tt}, \tag{3.13}$$

$$M_{pl}^2 G_{ti} = T^\phi_{ti}, \tag{3.14}$$

$$M_{pl}^2 G^i_j = T^\phi|^i_j, \tag{3.15}$$

with

$$G_{tt} = \frac{1}{2}\left({}^{[3]}R + \mathcal{K}^2 - \mathcal{K}^i_j\mathcal{K}^j_i\right), \tag{3.16}$$

$$G_{ti} = -\mathrm{D}_k\mathcal{K}^k_i + \mathrm{D}_i\mathcal{K}, \tag{3.17}$$

$$G^i_j = {}^{[3]}G^i_j - \dot{\mathcal{K}}^i_j + \mathcal{K}\mathcal{K}^i_j - \frac{1}{2}\delta^i_j\left(-2\dot{\mathcal{K}} + \mathcal{K}^2 + \mathcal{K}^k_l\mathcal{K}^l_k\right), \tag{3.18}$$

$$T^\phi_{tt} = K_X\dot{\phi}^2 - K - G_X\Box\phi\dot{\phi}^2 - G_X\dot{\phi}\dot{X} - G_X\partial_k X\partial^k\phi, \tag{3.19}$$

$$T^\phi_{ti} = K_X\dot{\phi}\partial_i\phi - G_X\Box\phi\dot{\phi}\partial_i\phi - G_X\dot{X}\partial_i\phi - G_X\partial_i X\dot{\phi}, \tag{3.20}$$

$$\begin{aligned} T^\phi|^i_j = {} & K_X\partial^i\phi\partial_j\phi - G_X\Box\phi\partial^i\phi\partial_j\phi - G_X\partial^i X\partial_j\phi - G_X\partial^i\phi\partial_j X \\ & + \left(G_X\partial^k\phi\partial_k X + K - G_X\dot{X}\dot{\phi}\right)\delta^i_j, \end{aligned} \tag{3.21}$$

where $\Box\phi = -\ddot{\phi} + \mathcal{K}\dot{\phi} + \mathrm{D}^\sigma\mathrm{D}_\sigma\phi$. The scalar equation of motion is given by

$$\partial_\mu J^\mu + \frac{1}{2}\partial_\mu \ln\gamma J^\mu = \dot{J}^t + (3H + 3\dot{\zeta})J^t + 3\partial_i\zeta J^i + \partial_i J^i = 0, \tag{3.22}$$

where $\gamma = \det[\gamma_{ij}]$ and $H = \dot{a}/a$ is the usual Hubble parameter.

4 Gradient expansion: order analysis

In standard cosmological perturbation theory one expands perturbatively in the field amplitudes. To tackle non-Gaussianities in inflation models, second order perturbation theory is often used within the Hubble horizon. However, for physics processes at superhorizon scales one usually resorts to the gradient expansion technique. Note that the separate universe approach or the δN formalism is simply the leading order gradient expansion [54]. Assuming the characteristic spatial length is L_{phys}, the dimensionless perturbative expansion parameter is

$$\epsilon \sim \frac{H^{-1}}{L_{phys}} \ll 1. \tag{4.1}$$

This means in particular that every spatial partial derivative carries an order of ϵ

$$\partial_i \sim \mathcal{O}(\epsilon), \tag{4.2}$$

while the time derivative is considered $\mathcal{O}(\epsilon^0)$. The superhorizon gradient expansion is complimentary to the usual non-linear cosmological perturbation analysis and may capture fully

nonlinear (in terms of the field amplitudes) physics at superhorizon scales, while the equations are still tractable due to the perturbative approach.

To perform the superhorizon perturbation analysis, we first need to deduce the starting orders for various quantities of interest. First, note that the equations of motion at $\mathcal{O}(\epsilon^0)$ should simply determine the evolution of the scale factor $a(t)$ and the scalar, as the spacetime is supposed to be described by an FLRW line element. Given the definition (3.6), one can infer that h_{ij} should start with $h_{ij}^{(0)}(\mathbf{x})$; otherwise the $\mathcal{O}(\epsilon^0)$ equation would pick up terms involving $\partial_t h_{ij}^{(0)}(t,\mathbf{x})$, which is non-FLRW. From the scalar's equation of motion eq. (3.22), we can infer that at $\mathcal{O}(\epsilon^0)$ the scalar field should be spatially homogeneous, meaning that ϕ starts with $\phi^{(0)}(t)$. Unlike previous work on this subject (see e.g. [24]), we do not impose any conditions on the higher orders of these quantities. Therefore, we have

$$\text{starting order of } \dot{h}_{ij} = \mathcal{O}(\epsilon), \tag{4.3}$$

$$\text{starting order of } \partial_i\phi = \mathcal{O}(\epsilon^2). \tag{4.4}$$

Expanding eq. (3.9) perturbatively (for $n \geq 1$)

$$\dot{h}_{ij}^{(n)} = -2\sum_{p=0}^{n-1} h_{ik}^{(p)} \left(A^{(n-p)}\right)^k_{\ j} \tag{4.5}$$

and making use of eq. (4.3), we can infer that

$$\text{starting order of } A_j^k = \mathcal{O}(\epsilon). \tag{4.6}$$

Expanding eq. (3.14), we infer that

$$\partial_i\mathcal{K}^{(0)} = 0. \tag{4.7}$$

Therefore, $\mathcal{K}^{(0)}$ is a function of t. From the definition (3.8), and given that one can always redefine the scalar factor $a(t)$ to absorb $\zeta^{(0)}(t)$, it follows that

$$\text{starting order of } \dot{\zeta} = \mathcal{O}(\epsilon), \tag{4.8}$$

$$\mathcal{K}^{(0)} = -3\frac{\dot{a}}{a} = -3H(t), \tag{4.9}$$

$$\mathcal{K}^{(n)} = -3\dot{\zeta}^{(n)}, \qquad n \geq 1, \tag{4.10}$$

where $H(t)$ is the usual Hubble parameter.

Using eq. (4.4), we may expand X as

$$X = X^{(0)}(t,\mathbf{x}) + X^{(1)}(t,\mathbf{x})\epsilon + X^{(2)}(t,\mathbf{x})\epsilon^2 + \mathcal{O}(\epsilon^3), \tag{4.11}$$

where

$$X^{(n)} = \frac{1}{2}\sum_{p=0}^{n}\dot{\phi}^{(p)}\dot{\phi}^{(n-p)} + \mathcal{O}(\epsilon^4). \tag{4.12}$$

We also need to perturbatively expand functions of X, such as $K(X)$. To this end, we should consider $X = X(\epsilon)$ according to eq. (4.11) and Taylor-expand, for example, $K(X(\epsilon))$ around $\epsilon = 0$ as

$$K(X) = K(X^{(0)}) + K_X(X^{(0)})X^{(1)}\epsilon + \frac{1}{2}\left[K_{XX}(X^{(0)})X^{(1)2} + 2K_X(X^{(0)})X^{(2)}\right]\epsilon^2 + \mathcal{O}(\epsilon^3). \tag{4.13}$$

In summary, the various quantities of interest, to be determined in the next section, are expanded as follows:

$$\zeta = \zeta^{(0)}(\mathbf{x}) + \zeta^{(1)}(t,\mathbf{x})\epsilon + \zeta^{(2)}(t,\mathbf{x})\epsilon^2 + \mathcal{O}(\epsilon^3), \tag{4.14}$$

$$\phi = \phi^{(0)}(t) + \phi^{(1)}(t,\mathbf{x})\epsilon + \phi^{(2)}(t,\mathbf{x})\epsilon^2 + \mathcal{O}(\epsilon^3)\,, \tag{4.15}$$

$$A^i_j = A^{(1)i}{}_j(t,\mathbf{x})\epsilon + A^{(2)i}{}_j(t,\mathbf{x})\epsilon^2 + \mathcal{O}(\epsilon^3), \tag{4.16}$$

$$h_{ij} = h^{(0)}_{ij}(\mathbf{x}) + h^{(1)}_{ij}(t,\mathbf{x})\epsilon + h^{(2)}_{ij}(t,\mathbf{x})\epsilon^2 + \mathcal{O}(\epsilon^3)\,, \tag{4.17}$$

$$\mathcal{K}^i_j = -H(t)\delta^i_j + \mathcal{K}^{(1)i}{}_j(t,\mathbf{x})\epsilon + \mathcal{K}^{(2)i}{}_j(t,\mathbf{x})\epsilon^2 + \mathcal{O}(\epsilon^3), \tag{4.18}$$

$$\mathcal{K} = -3H(t) + \dot\zeta^{(1)}(t,\mathbf{x})\epsilon + \dot\zeta^{(2)}(t,\mathbf{x})\epsilon^2 + \mathcal{O}(\epsilon^3). \tag{4.19}$$

5 General solution

Now, we solve the equations of motion perturbatively, up to the $\mathcal{O}(\epsilon^2)$ order, to obtain the general solutions. These solutions will be parametrized by a few unspecified spatial functions, which describe the physical degrees of freedom (modulo residual gauge freedom) that may evolve as the Universe expands. The gradient expansions of Einstein's equations up to order $\mathcal{O}(\epsilon^2)$ are listed in Appendix B.

For the rest of this paper, to simplify the equations, we will mostly suppress the background quantities' order indication $^{(0)}$. For example, $\dot\phi^{(0)}$ is written as $\dot\phi$ if there is no confusion.

5.1 The $\mathcal{O}(\epsilon^0)$ order

For the $\mathcal{O}(\epsilon^0)$ order, all spatial derivatives are absent. As desired, the equations of motion reduced to the conventional background FLRW case:

$$3M_{pl}^2H^2 = -K + 2K_XX + 6H\dot\phi G_XX, \tag{5.1}$$

$$-M_{pl}^2\left(2\dot H + 3H^2\right) = K - 2G_XX\ddot\phi, \tag{5.2}$$

$$\dot J^{t(0)} + 3HJ^{t(0)} = 0, \tag{5.3}$$

where $J^{t(0)} = K_X\dot\phi + 6HG_XX$. Note that only two of the three equations are independent.

5.2 The $\mathcal{O}(\epsilon)$ order

The tt component of Einstein's equation is given by

$$\left(\frac{1}{2}\dot\phi K_X + \dot\phi K_{XX}X + 9HG_XX + 6HG_{XX}X^2\right)\dot\phi^{(1)} = \left(3M_{pl}^2H - 3\dot\phi G_XX\right)\dot\zeta^{(1)}, \tag{5.4}$$

which can be re-written as

$$\dot\phi^{(1)} = \mathcal{A}^0\dot\zeta^{(1)}, \tag{5.5}$$

where

$$\mathcal{A}^0(t) = \frac{6M_{pl}^2H - 6\dot\phi G_XX}{\dot\phi K_X + 2\dot\phi K_{XX}X + 18HG_XX + 12HG_{XX}X^2}. \tag{5.6}$$

The ij component of Einstein's equation naturally splits into a trace part and a traceless part. The trace part gives rise to another relation between $\phi^{(1)}$ and $\zeta^{(1)}$:

$$-2G_XX\ddot\phi^{(1)} + \left(\dot\phi K_X - 2\dot X(G_X + G_{XX}X)\right)\dot\phi^{(1)} + 2M_{pl}^2\left(\ddot\zeta^{(1)} + 3H\dot\zeta^{(1)}\right) = 0. \tag{5.7}$$

Combining eq. (5.5) and eq. (5.7), we get, after integration,

$$\zeta^{(1)}(t,\mathbf{x}) = C_{\zeta}^{(1)}(\mathbf{x}) \int^{t} \frac{\mathrm{d}t'}{\bar{a}(t')^3}, \tag{5.8}$$

where

$$\bar{a}(t) = \exp(\int^{t} \mathrm{d}t' \mathcal{H}^0(t')), \tag{5.9}$$

$$\mathcal{H}^0(t) = \frac{\left(\dot{\phi}K_X - 2\dot{X}(G_X + G_{XX}X)\right)\mathcal{A}^0 - 2G_X X\dot{\mathcal{A}}^0 + 6M_{pl}^2 H}{6M_{pl}^2 - 6G_X X\mathcal{A}^0}, \tag{5.10}$$

and $\mathcal{C}_{\zeta}^{(1)}(\mathbf{x})$ is an unspecified spatial function from the first integration. There would be another unspecified spatial function from the second integration ($C_{\zeta}'^{(1)}(\mathbf{x})$), which has been absorbed into $\zeta^{(0)}(\mathbf{x})$. We will see in the next section that $\mathcal{H}^0$ approaches the Hubble constant H for near de Sitter expansion, in which case $\dot{\zeta}^{(1)}$ scales as $1/a^3(t)$. Now, $\phi^{(1)}$ is given by

$$\phi^{(1)}(t,\mathbf{x}) = C_{\zeta}^{(1)}(\mathbf{x}) \int^{t} \frac{\mathrm{d}t'\mathcal{A}^0(t')}{\bar{a}(t')^3} + C_{\phi}^{(1)}(\mathbf{x}), \tag{5.11}$$

where $C_{\phi}^{(1)}(\mathbf{x})$ is an integration spatial function. The traceless part of Einstein's equation's ij component is simply

$$\dot{A}^{(1)i}{}_{j} + 3HA^{(1)i}{}_{j} = 0, \tag{5.12}$$

whose solution is

$$A^{(1)i}{}_{j}(t,\mathbf{x}) = \frac{C_{A}^{(1)i}{}_{j}(\mathbf{x})}{a^3}, \tag{5.13}$$

where the unspecified spatial function $C_{A}^{(1)i}{}_{j}(\mathbf{x})$ is symmetric and traceless. From eq. (3.9), we have

$$h_{ij}^{(1)}(t,\mathbf{x}) = -2h_{ik}^{(0)}(\mathbf{x})C_{A}^{(1)k}{}_{j}(\mathbf{x}) \int^{t} \frac{\mathrm{d}t'}{a(t')^3}, \tag{5.14}$$

where the would-be integration spatial function $C_{h}^{(1)}{}_{ij}(\mathbf{x})$ has been absorbed into $h_{ik}^{(0)}(\mathbf{x})$. As expected, the scalar equation of motion is identically the solution obtained above.

Before moving on to solve higher order equations, we note that defining a curvature perturbation that is conserved in time is trivial in our formalism. By virtue of the tt component of Einstein's equation (5.5), one can define a conserved curvature perturbation at $\mathcal{O}(\epsilon)$

$$\mathcal{R}^{(1)} = \zeta^{(1)} - \int^{t} \frac{\mathrm{d}t'}{\mathcal{A}^0(t')}\dot{\phi}^{(1)}(t'). \tag{5.15}$$

As we will see in Section 6, because of the shift symmetry, de Sitter expansion is an attractor of the system. For quasi-de Sitter expansion, i.e., for the late time of Galileon inflation, $\mathcal{A}^0 \simeq$ constant and we can write $\mathcal{R}^{(1)}$ as

$$\mathcal{R}^{(1)} \simeq \zeta^{(1)} - \frac{1}{\mathcal{A}^0}\phi^{(1)}. \tag{5.16}$$

5.3 The $\mathcal{O}(\epsilon^2)$ order

The tt component of the Einstein equation gives

$$\mathcal{A}^0\dot{\zeta}^{(2)} - \dot{\phi}^{(2)} = \mathcal{C}^0\left(\dot{\zeta}^{(1)}\right)^2 - \frac{\mathcal{C}_3^0}{2}\,{}^{[3]}R^{(2)} + \frac{\mathcal{C}_3^0}{2}A^{(1)k}_{\ \ \ j}A^{(1)j}_{\ \ \ k}, \tag{5.17}$$

where ${}^{[3]}R^{(2)}$ is the 3D Ricci scalar for the $\mathcal{O}(\epsilon^0)$ order metric $\gamma_{ij} = a(t)^2 e^{2\zeta^{(0)}(\mathbf{x})}h_{ij}^{(0)}(\mathbf{x})$ and $\mathcal{C}^0$, $\mathcal{C}_1^0$, $\mathcal{C}_2^0$ and $\mathcal{C}_3^0$ are again background quantities, defined respectively as

$$\mathcal{C}^0(t) = \mathcal{C}_1^0(\mathcal{A}^0)^2 + \mathcal{C}_2^0\mathcal{A}^0 - 3\mathcal{C}_3^0, \tag{5.18}$$

$$\mathcal{C}_1^0(t) = \frac{\frac{1}{2}K_X + 4K_{XX}X + 2K_{XXX}X^2 + 9H\dot{\phi}G_X + 21H\dot{\phi}G_{XX}X + 6H\dot{\phi}G_{XXX}X^2}{\dot{\phi}K_X + 2\dot{\phi}K_{XX}X + 18HG_XX + 12HG_{XX}X^2}, \tag{5.19}$$

$$\mathcal{C}_2^0(t) = \frac{18G_XX + 12G_{XX}X^2}{\dot{\phi}K_X + 2\dot{\phi}K_{XX}X + 18HG_XX + 12HG_{XX}X^2}, \tag{5.20}$$

$$\mathcal{C}_3^0(t) = \frac{M_{pl}^2}{\dot{\phi}K_X + 2\dot{\phi}K_{XX}X + 18HG_XX + 12HG_{XX}X^2}. \tag{5.21}$$

Integrating this equation, we get the solution of $\phi^{(2)}$ in terms of $\zeta^{(2)}$:

$$\begin{aligned}\phi^{(2)}(t,\mathbf{x}) = &\int^t \mathrm{d}t'\mathcal{A}^0(t')\dot{\zeta}^{(2)}(t',\mathbf{x}) - \left(\mathcal{C}_\zeta^{(1)}(\mathbf{x})\right)^2\int^t \frac{\mathrm{d}t'\mathcal{C}^0(t')}{\bar{a}(t')^6} + \frac{{}^{[3]}\bar{R}^{(2)}(\mathbf{x})}{2}\int^t\frac{\mathrm{d}t'\mathcal{C}_3^0(t')}{a(t')^2}\\ &- \frac{C_{A\ \ j}^{(1)k}(\mathbf{x})C_{A\ \ k}^{(1)j}(\mathbf{x})}{2}\int^t\frac{\mathrm{d}t'\mathcal{C}_3^0(t')}{a(t')^6},\end{aligned} \tag{5.22}$$

where an integration spatial function has been absorbed into $C_\phi^{(1)}(\mathbf{x})$, and ${}^{[3]}\bar{R}^{(2)}(\mathbf{x})$ (the Ricci scalar of the metric $e^{2\zeta^{(0)}(\mathbf{x})}h_{ij}^{(0)}(\mathbf{x})$) is related to ${}^{[3]}R^{(2)}$ (the Ricci scalar of the metric $a(t)^2e^{2\zeta^{(0)}(\mathbf{x})}h_{ij}^{(0)}(\mathbf{x})$) by

$${}^{[3]}\bar{R}^{(2)}(\mathbf{x}) = a(t)^2\,{}^{[3]}R^{(2)}. \tag{5.23}$$

The trace part of Einstein's equation's ij component is given by

$$\begin{aligned}&-M_{pl}^2\left(2\ddot{\zeta}^{(2)} + 6H\dot{\zeta}^{(2)} + 3\left(\dot{\zeta}^{(1)}\right)^2 + \frac{1}{2}A^{(1)k}_{\ \ \ l}A^{(1)l}_{\ \ \ k} + \frac{1}{6}\,{}^{[3]}R^{(2)}\right)\\ &= -2G_XX\ddot{\phi}^{(2)} + \left(\dot{\phi}K_X - 2\dot{X}(G_X + G_{XX}X)\right)\dot{\phi}^{(2)} + \mathcal{D}^0\left(\dot{\phi}^{(1)}\right)^2,\end{aligned} \tag{5.24}$$

where

$$\begin{aligned}\mathcal{D}^0(t) = \Big(&\frac{1}{2}K_X + K_{XX}X - \ddot{\phi}G_X - 5\ddot{\phi}G_{XX}X - 2\ddot{\phi}G_{XXX}X^2\\ &+ 2\left(3\mathcal{H}^0 - \partial_t\ln\mathcal{A}^0\right)\dot{\phi}(G_X + G_{XX}X)\Big).\end{aligned} \tag{5.25}$$

Combining with eq. (5.17), we get

$$\begin{aligned}\zeta^{(2)}(t,\mathbf{x}) = &\left(C_\zeta^{(1)}(\mathbf{x})\right)^2\int^t\frac{\mathrm{d}t''}{\bar{a}(t'')^3}\int^{t''}\frac{\mathrm{d}t'\mathcal{E}_1^0(t')}{\bar{a}(t')^3} + {}^{[3]}\bar{R}^{(2)}(\mathbf{x})\int^t\frac{\mathrm{d}t''}{\bar{a}(t'')^3}\int^{t''}\frac{\mathrm{d}t'\mathcal{E}_3^0(t')\bar{a}(t')^3}{a(t')^2}\\ &+ C_{A\ \ l}^{(1)k}(\mathbf{x})C_{A\ \ k}^{(1)l}(\mathbf{x})\int^t\frac{\mathrm{d}t''}{\bar{a}(t'')^3}\int^{t''}\frac{\mathrm{d}t'\mathcal{E}_2^0(t')\bar{a}(t')^3}{a(t')^6},\end{aligned} \tag{5.26}$$

where two integration spatial functions have been absorbed into $C_\zeta^{(1)}(\mathbf{x})$ and $\zeta^{(0)}(\mathbf{x})$ respectively, and $\mathcal{E}_1^0$, $\mathcal{E}_2^0$ and $\mathcal{E}_3^0$ are background quantities, defined respectively as

$$\mathcal{E}_1^0(t) = \frac{\left(\dot{\phi}K_X - 2\dot{X}(G_X + G_{XX}X)\right)\mathcal{C}^0 + 2G_X X(6\mathcal{H}^0\mathcal{C}^0 - \dot{\mathcal{C}}^0) - \mathcal{D}^0\left(\mathcal{A}^0\right)^2 - 3M_{pl}^2}{2M_{pl}^2 - 2G_X X\mathcal{A}^0}, \quad (5.27)$$

$$\mathcal{E}_2^0(t) = \frac{\left(\dot{\phi}K_X - 2\dot{X}(G_X + G_{XX}X)\right)\mathcal{C}_3^0 + 2G_X X(6H\mathcal{C}_3^0 - \dot{\mathcal{C}}_3^0) - M_{pl}^2}{4M_{pl}^2 - 4G_X X\mathcal{A}^0}, \quad (5.28)$$

$$\mathcal{E}_3^0(t) = -\frac{\left(\dot{\phi}K_X - 2\dot{X}(G_X + G_{XX}X)\right)\mathcal{C}_3^0 + 2G_X X(2H\mathcal{C}_3^0 - \dot{\mathcal{C}}_3^0) + \frac{1}{3}M_{pl}^2}{4M_{pl}^2 - 4G_X X\mathcal{A}^0}. \quad (5.29)$$

The traceless part of Einstein's equation's ij component is given by

$$\dot{A}^{(2)i}{}_j + 3HA^{(2)i}{}_j + 3\dot{\zeta}^{(1)}A^{(1)i}{}_j - \left({}^{[3]}R^{(2)i}{}_j - \frac{1}{3}\delta^i_j\,{}^{[3]}R^{(2)}\right) = 0, \quad (5.30)$$

which gives rise to the solution

$$A^{(2)i}{}_j(t,\mathbf{x}) = -\frac{3C_\zeta^{(1)}(\mathbf{x})C_A^{(1)i}{}_j(\mathbf{x})}{a^3}\int^t \frac{\mathrm{d}t'}{\bar{a}(t')^3} + \frac{{}^{[3]}\mathcal{R}^{(2)i}{}_j(\mathbf{x}) - \frac{1}{3}\delta^i_j{}^{[3]}\mathcal{R}^{(2)}(\mathbf{x})}{a^3}\int^t \mathrm{d}t' a(t'), \quad (5.31)$$

where again an integration spatial function has been absorbed into $C_A^{(1)k}{}_j(\mathbf{x})$ and ${}^{[3]}\mathcal{R}^{(2)i}{}_j(\mathbf{x})$ (the Ricci tensor of the metric $e^{2\zeta^{(0)}(\mathbf{x})}h_{ij}^{(0)}(\mathbf{x})$) is related to ${}^{[3]}R^{(2)i}{}_j$ (the Ricci tensor of the metric $a(t)^2e^{2\zeta^{(0)}(\mathbf{x})}h_{ij}^{(0)}(\mathbf{x})$) by

$${}^{[3]}\mathcal{R}^{(2)i}{}_j(\mathbf{x}) = a(t)^2\,{}^{[3]}R^{(2)i}{}_j. \quad (5.32)$$

From eq. (3.9), we can derive

$$\begin{aligned}
h_{ij}^{(2)}(t,\mathbf{x}) = {}& 6h_{ik}^{(0)}(\mathbf{x})C_\zeta^{(1)}(\mathbf{x})C_A^{(1)k}{}_j(\mathbf{x})\int^t \frac{\mathrm{d}t''}{a(t'')^3}\int^{t''}\frac{\mathrm{d}t'}{\bar{a}(t')^3} \\
&+ 4h_{il}^{(0)}(\mathbf{x})C_A^{(1)l}{}_k(\mathbf{x})C_A^{(1)k}{}_j(\mathbf{x})\int^t \frac{\mathrm{d}t''}{a(t'')^3}\int^{t''}\frac{\mathrm{d}t'}{a(t')^3} \\
&- 2h_{ik}^{(0)}(\mathbf{x})\left({}^{[3]}\mathcal{R}^{(2)k}{}_j(\mathbf{x}) - \frac{1}{3}\delta^k_j{}^{[3]}\mathcal{R}^{(2)}(\mathbf{x})\right)\int^t \frac{\mathrm{d}t''}{a(t'')^3}\int^{t''}\mathrm{d}t' a(t'),
\end{aligned} \quad (5.33)$$

where an integration spatial function has been absorbed into $h^{(0)}{}_{ij}(\mathbf{x})$. The ti component of Einstein's equation at the $\mathcal{O}(\epsilon^2)$ order become constraints for the $\mathcal{O}(\epsilon)$ order quantities

$$-2M_{pl}^2\partial_i\dot{\zeta}^{(1)} - M_{pl}^2\mathrm{D}_k^{(1)}A^{(1)k}{}_i = (K_X\dot{\phi} + 6HG_X X)\partial_i\phi^{(1)} - 2G_X X\partial_i\dot{\phi}^{(1)}, \quad (5.34)$$

where $\mathrm{D}_k^{(1)}$, of order $\mathcal{O}(\epsilon)$ itself, is the covariant derivative associated with the $\mathcal{O}(\epsilon^0)$ order metric $e^{2\zeta^{(0)}(\mathbf{x})}h_{ij}^{(0)}(\mathbf{x})$. This gives rise to 3 constraints on the unspecified integration functions $C_A^{(1)i}{}_j(\mathbf{x})$.

5.4 Summary

Here we summarize the solution obtained up to the $\mathcal{O}(\epsilon^2)$ order:

$$
\begin{aligned}
\zeta(t,\mathbf{x}) = {}& \zeta^{(0)}(\mathbf{x}) + C_\zeta^{(1)}(\mathbf{x})\int^t \frac{\mathrm{d}t'}{\bar{a}(t')^3} + \left(C_\zeta^{(1)}(\mathbf{x})\right)^2 \int^t \frac{\mathrm{d}t''}{\bar{a}(t'')^3}\int^{t''}\frac{\mathrm{d}t'\mathcal{E}_1^0(t')}{\bar{a}(t')^3} \\
&+ C_A^{(1)}{}^k_l(\mathbf{x}) C_A^{(1)}{}^l_k(\mathbf{x}) \int^t \frac{\mathrm{d}t''}{\bar{a}(t'')^3}\int^{t''}\frac{\mathrm{d}t'\mathcal{E}_2^0(t')\bar{a}(t')^3}{a(t')^6} \\
&+ {}^{[3]}R^{(2)}(\mathbf{x}) \int^t \frac{\mathrm{d}t''}{\bar{a}(t'')^3}\int^{t''}\frac{\mathrm{d}t'\mathcal{E}_3^0(t')\bar{a}(t')^3}{a(t')^2} + \mathcal{O}(\epsilon^3),
\end{aligned} \tag{5.35}
$$

$$
\begin{aligned}
\phi(t,\mathbf{x}) = {}& \phi^{(0)}(t) + C_\phi^{(1)}(\mathbf{x}) + C_\zeta^{(1)}(\mathbf{x})\int^t \frac{\mathrm{d}t'\mathcal{A}^0(t')}{\bar{a}(t')^3} + \int^t \mathrm{d}t'\mathcal{A}^0(t')\dot{\zeta}^{(2)}(t',\mathbf{x}) \\
&- \left(C_\zeta^{(1)}(\mathbf{x})\right)^2 \int^t \frac{\mathrm{d}t'\mathcal{C}^0(t')}{\bar{a}(t')^6} + \frac{{}^{[3]}R^{(2)}(\mathbf{x})}{2}\int^t \frac{\mathrm{d}t'\mathcal{C}_3^0(t')}{a(t')^2} \\
&- \frac{C_A^{(1)}{}^k_j(\mathbf{x}) C_A^{(1)}{}^j_k(\mathbf{x})}{2}\int^t \frac{\mathrm{d}t'\mathcal{C}_3^0(t')}{a(t')^6} + \mathcal{O}(\epsilon^3),
\end{aligned} \tag{5.36}
$$

$$
\begin{aligned}
A^i_j(t,\mathbf{x}) = {}& \frac{C_A^{(1)}{}^i_j(\mathbf{x})}{a^3} - \frac{3C_\zeta^{(1)}(\mathbf{x})C_A^{(1)}{}^i_j(\mathbf{x})}{a^3}\int^t \frac{\mathrm{d}t'}{\bar{a}(t')^3} \\
&+ \frac{{}^{[3]}R^{(2)}{}^i_j(\mathbf{x}) - \frac{1}{3}\delta^i_j{}^{[3]}R^{(2)}(\mathbf{x})}{a^3}\int^t \mathrm{d}t' a(t') + \mathcal{O}(\epsilon^3),
\end{aligned} \tag{5.37}
$$

$$
\begin{aligned}
h_{ij}(t,\mathbf{x}) = {}& h_{ij}^{(0)}(\mathbf{x}) - 2h_{ik}^{(0)}(\mathbf{x})C_A^{(1)}{}^k_j(\mathbf{x})\int^t \frac{\mathrm{d}t'}{a(t')^3} \\
&+ 6h_{ik}^{(0)}(\mathbf{x})C_\zeta^{(1)}(\mathbf{x})C_A^{(1)}{}^k_j(\mathbf{x})\int^t \frac{\mathrm{d}t''}{a(t'')^3}\int^{t''}\frac{\mathrm{d}t'}{\bar{a}(t')^3} \\
&+ 4h_{il}^{(0)}(\mathbf{x})C_A^{(1)}{}^l_k(\mathbf{x})C_A^{(1)}{}^k_j(\mathbf{x})\int^t \frac{\mathrm{d}t''}{a(t'')^3}\int^{t''}\frac{\mathrm{d}t'}{a(t')^3} \\
&- 2h_{ik}^{(0)}(\mathbf{x})\left({}^{[3]}R^{(2)}{}^k_j(\mathbf{x}) - \frac{1}{3}\delta^k_j{}^{[3]}R^{(2)}(\mathbf{x})\right)\int^t \frac{\mathrm{d}t''}{a(t'')^3}\int^{t''}\mathrm{d}t' a(t'),
\end{aligned} \tag{5.38}
$$

where $\mathcal{A}^0$ is defined by eq. (5.6), $\bar{a}(t)$ is defined by eq. (5.9), $\mathcal{C}^0$, $\mathcal{C}_1^0$, $\mathcal{C}_2^0$ and $\mathcal{C}_3^0$ are defined by eqs. (5.18-5.21) respectively, $\mathcal{E}_1^0$, $\mathcal{E}_2^0$ and $\mathcal{E}_3^0$ are defined by eqs. (5.27-5.29) respectively, ${}^{[3]}R^{(2)}{}^i_j(\mathbf{x})$ and ${}^{[3]}R^{(2)}(\mathbf{x})$ are 3D curvature tensors of the metric $e^{2\zeta^{(0)}(\mathbf{x})}h_{ij}^{(0)}(\mathbf{x})$.

There are several unspecified spatial functions in the general solution: $\zeta^{(0)}(\mathbf{x})$, $h_{ij}^{(0)}(\mathbf{x})$, $C_\zeta^{(1)}(\mathbf{x})$, $C_\phi^{(1)}(\mathbf{x})$ and $C_A^{(1)}{}^i_j(\mathbf{x})$. (There are a few other $\mathcal{O}(\epsilon)$ and $\mathcal{O}(\epsilon^2)$ unspecified spatial functions, which have been absorbed into these unspecified functions.) $h_{ij}^{(0)}(\mathbf{x})$ is symmetric and has a unit determinant and $C_A^{(1)}{}^i_j(\mathbf{x})$ is symmetric and traceless, so they each have 5 degrees of freedom. 3 components of $C_A^{(1)}{}^i_j(\mathbf{x})$ are related to other unspecified spatial functions respectively by the constraint equations (5.34).

To count the physical degrees of freedom we should identify the residual gauge freedom. Performing the infinitesimal coordinate transformation

$$x^\mu \to \bar{x}^\mu = x^\mu + \eta^\mu \tag{5.39}$$

and requiring that the synchronous gauge condition on the lapse N and shift N^i are respected, one straighforwadly obtains that η^μ should be of the form

$$\eta^0 = \eta^0(\mathbf{x}) \tag{5.40}$$

$$\eta^i = \int^t \mathrm{d}t' \gamma^{ij}(t', \mathbf{x}) \partial_j \eta^0(\mathbf{x}) + \tilde{\eta}^i(\mathbf{x}) \tag{5.41}$$

(η^μ may be chosen as $\mathcal{O}(\epsilon)$). From this we infer that the residual gauge freedom amounts to 4 functions of space. 3 of those can be chosen so as to eliminate 3 spatial functions in $h_{ij}^{(0)}(\mathbf{x})$ and 1 chosen so as to eliminate $C_\phi^{(1)}(\mathbf{x})$. Therefore, we may count the degrees of freedoms as follows:

$$\zeta^{(0)}(\mathbf{x}) \qquad \text{1 scalar growing mode} \ = \ \text{1 component}, \tag{5.42}$$

$$h_{ij}^{(0)}(\mathbf{x}) \qquad \text{2 tensor growing modes} \ = \ \text{5 components} \ - \ \text{3 gauge DoFs}, \tag{5.43}$$

$$C_\zeta^{(1)}(\mathbf{x}) \qquad \text{1 scalar decaying mode} \ = \ \text{1 component}, \tag{5.44}$$

$$C_{A\ \, j}^{(1)i}(\mathbf{x}) \qquad \text{2 tensor decaying modes} \ = \ \text{5 components} \ - \ \text{3 constraints}. \tag{5.45}$$

6 Late time of Galileon inflation

At the end of the last section, we claimed that $C_\zeta^{(1)}(\mathbf{x})$ represents a decaying mode. However, this is actually not obvious from the general solution given above. After all, $\bar{a}(t)$ is not the scale factor $a(t)$ but is given by a rather complicated expression in terms of background quantities. Additionally, $\mathcal{C}_n^0$ and $\mathcal{E}_n^0$ also have time dependence. In this section, we would like to briefly re-derive the solution for an important special case, the late time of Galileon inflation. This will not only allows us to show explicitly that $C_\zeta^{(1)}(\mathbf{x})$ represents a decaying mode, but it will demonstrate how one can eliminate the gauge mode $C_\phi^{(1)}(\mathbf{x})$ on inflationary backgrounds. Moreover, the assumption of quasi-de Sitter expansion drastically simplifies the solution and allows an intuitive understanding of its behaviour. Physically, perturbations coming from the late time of inflation are observationally most important, as it is these perturbations that seed the large scale structure of the observable Universe. Note that, similar to the previous section, we mostly suppress the background quantities' order indication $^{(0)}$ to simplify the equations.

Eq. (5.3) can be integrated to get

$$K_X \dot{\phi} + 6HG_X X \propto a(t)^{-3}, \tag{6.1}$$

which is an attractor of the dynamical system. So for the later time of Galileon inflation J^t essentially vanishes. In this limit, the background equations of motion can be simplified to

$$K = -3M_{pl}^2 H^2, \tag{6.2}$$

$$K_X = -3G_X H \dot{\phi}, \tag{6.3}$$

and $\dot{\phi}$ and H become constant and $a \propto e^{Ht}$ [35]. Furthermore, we have

$$\mathcal{H}^0 \to H, \qquad \bar{a}(t) \to a(t). \tag{6.4}$$

The background quantities defined in the last section now all become constant and can also be simplified:

$$\mathcal{A}^0 = \frac{3M_{pl}^2 H - 3\dot{\phi} G_X X}{\dot{\phi} K_{XX} X + 6HG_X X + 6HG_{XX} X^2}, \tag{6.5}$$

$$\mathcal{C}^0 = \mathcal{C}_1^0 (\mathcal{A}^0)^2 + \mathcal{C}_2^0 \mathcal{A}^0 - 3\mathcal{C}_3^0, \tag{6.6}$$

$$\mathcal{C}_1^0 = \frac{-\frac{5}{2}K_X + 4K_{XX}X + 2K_{XXX}X^2 + 21H\dot{\phi}G_{XX}X + 6H\dot{\phi}G_{XXX}X^2}{2\dot{\phi}K_{XX}X + 12HG_X X + 12HG_{XX}X^2}, \tag{6.7}$$

$$\mathcal{C}_2^0 = \frac{9G_X X + 6G_{XX}X^2}{\dot{\phi}K_{XX}X + 6HG_X X + 6HG_{XX}X^2}, \tag{6.8}$$

$$\mathcal{C}_3^0 = \frac{M_{pl}^2}{2\dot{\phi}K_{XX}X + 12HG_X X + 12HG_{XX}X^2}, \tag{6.9}$$

$$\mathcal{D}^0 = -\frac{3}{2}K_X + K_{XX}X + 6H\dot{\phi}G_{XX}X, \tag{6.10}$$

$$\mathcal{E}_1^0 = -\frac{\dot{\phi}K_X\mathcal{C}^0 + \mathcal{D}^0\left(\mathcal{A}^0\right)^2 + 3M_{pl}^2}{2M_{pl}^2 - 2G_X X\mathcal{A}^0}, \tag{6.11}$$

$$\mathcal{E}_2^0 = -\frac{\dot{\phi}K_X\mathcal{C}_3^0 + M_{pl}^2}{4M_{pl}^2 - 4G_X X\mathcal{A}^0}, \tag{6.12}$$

$$\mathcal{E}_3^0 = \frac{1}{3}\mathcal{E}_2^0. \tag{6.13}$$

Note that we have assumed $\dot{\phi}K_{XX}X + 6HG_X X + 6HG_{XX}X^2 \neq 0$ and $\mathcal{A}^0 + 6M_{pl}^2 H/\dot{\phi}K_X \neq 0$, which, by using the background EoMs, is equivalent to $G_X(K_X - K_{XX}X) + K_X G_{XX}X \neq 0$ and $K(G_X K_X - 2G_X K_{XX}X + 2K_X G_{XX}X) + K_X^2 G_X X \neq 0$. So the covariant cubic Galileon case is included in our solution. We will not discuss the special cases where any of the aforementioned quantities actually vanish, but it is easy to follow our formalism in the last section to get the relevant results. The constraint eq. (5.34) now becomes

$$2(G_X X\mathcal{A}^0 - M_{pl}^2)\partial_i C_\zeta^{(1)}(\mathbf{x}) = M_{pl}^2 \mathrm{D}_k^{(1)} C_A^{(1)}{}^k_{\ i}(\mathbf{x}). \tag{6.14}$$

Finally, the solution for the late time of Galileon inflation is given by

$$\begin{aligned}\zeta(t,\mathbf{x}) = \zeta^{(0)}(\mathbf{x}) &- \frac{C_\zeta^{(1)}(\mathbf{x})}{3Ha^3} + \frac{\mathcal{E}_1^0\left(C_\zeta^{(1)}(\mathbf{x})\right)^2}{18H^2a^6} + \frac{\mathcal{E}_2^0 C_A^{(1)}{}^k_{\ l}(\mathbf{x})C_A^{(1)}{}^l_{\ k}(\mathbf{x})}{18H^2a^6} \\ &- \frac{\mathcal{E}_3^0{}^{[3]}R^{(2)}(\mathbf{x})}{2H^2a^2} + \mathcal{O}(\epsilon^3),\end{aligned} \tag{6.15}$$

$$\begin{aligned}\phi(t,\mathbf{x}) = \phi^{(0)}(t) + C_\phi^{(1)}(\mathbf{x}) &- \frac{\mathcal{A}^0 C_\zeta^{(1)}(\mathbf{x})}{3Ha^3} + \mathcal{A}^0\zeta^{(2)}(t,\mathbf{x}) + \frac{\mathcal{C}^0\left(C_\zeta^{(1)}(\mathbf{x})\right)^2}{6Ha^6} \\ &- \frac{\mathcal{C}_3^0\,{}^{[3]}R^{(2)}(\mathbf{x})}{4Ha^2} + \frac{\mathcal{C}_3^0 C_A^{(1)}{}^k_{\ j}(\mathbf{x})C_A^{(1)}{}^j_{\ k}(\mathbf{x})}{12Ha^6} + \mathcal{O}(\epsilon^3),\end{aligned} \tag{6.16}$$

$$A^i_j(t,\mathbf{x}) = \frac{C_A^{(1)}{}^i_{\ j}(\mathbf{x})}{a^3} + \frac{C_\zeta^{(1)}(\mathbf{x})C_A^{(1)}{}^i_{\ j}(\mathbf{x})}{Ha^6} + \frac{{}^{[3]}R^{(2)}{}^i_{\ j}(\mathbf{x}) - \frac{1}{3}\delta^i_j{}^{[3]}R^{(2)}(\mathbf{x})}{Ha^2} + \mathcal{O}(\epsilon^3), \tag{6.17}$$

$$h_{ij}(t,\mathbf{x}) = h_{ij}^{(0)}(\mathbf{x}) + \frac{2h_{ik}^{(0)}(\mathbf{x})C_A^{(1)}{}^k_{\ j}(\mathbf{x})}{3Ha^3} + \frac{2h_{ik}^{(0)}(\mathbf{x})\left({}^{[3]}R^{(2)}{}^k_{\ j}(\mathbf{x}) - \frac{1}{3}\delta^k_j{}^{[3]}R^{(2)}(\mathbf{x})\right)}{3H^2a^2}$$

$$+\frac{3h^{(0)}_{ik}(\mathbf{x})C^{(1)}_{\zeta}(\mathbf{x})C^{(1)}_{A}{}^{k}_{\ j}(\mathbf{x})+2h^{(0)}_{il}(\mathbf{x})C^{(1)}_{A}{}^{l}_{\ k}(\mathbf{x})C^{(1)}_{A}{}^{k}_{\ j}(\mathbf{x})}{9H^2a^6}+\mathcal{O}(\epsilon^3). \tag{6.18}$$

Now, we want to explicitly do away with the gauge mode $C^{(1)}_{\phi}(\mathbf{x})$ in the case of near de Sitter inflation by re-slicing. To this end, we choose

$$\bar{t}=t+\eta^0(\mathbf{x}), \tag{6.19}$$
$$\bar{x}^i=x^i+\eta^i, \tag{6.20}$$

with

$$\eta^0(\mathbf{x})=\frac{C^{(1)}_{\phi}(\mathbf{x})}{\dot{\phi}^{(0)}}, \tag{6.21}$$

$$\eta^i=\int^t \mathrm{d}t'\gamma^{ij}(t',\mathbf{x})\partial_j\eta^0(\mathbf{x})=-\frac{h^{ij}_{(0)}(\mathbf{x})\partial_jC^{(1)}_{\phi}(\mathbf{x})}{2H\dot{\phi}^{(0)}e^{2\zeta^{(0)}(\mathbf{x})}a(\bar{t})^2}+\mathcal{O}(\epsilon^3) \tag{6.22}$$

Let us consider the effects of the temporal transformation on $\phi^{(0)}(t)$: Taylor expansion yields $\phi^{(0)}(t)=\phi^{(0)}(\bar{t})-C^{(1)}_{\phi}(\mathbf{x})+\mathcal{O}(\epsilon^3)$, which straightforwardly removes the constant mode in eq. (6.16). Though far less obvious, any other effect of the temporal or the spatial part of he transformation leads to corrections that are either $\mathcal{O}(\epsilon^3)$ or can be absorbed in redefinitions of $C^{(1)}_{\zeta}(\mathbf{x})$ and $C^{(1)}_{A}{}^{i}_{\ j}(\mathbf{x})$. The end result is that by re-slicing one can eliminate $C^{(1)}_{\phi}(\mathbf{x})$ with all the other terms in the solution unchanged.

7 Conclusion

Non-Gaussianity in primordial perturbations can be a powerful probe of different inflation models and the associated fundamental theory on which they are based. To calculate or evolve non-Gaussian perturbations, one has to go beyond linear perturbation theory. Complementary to second order cosmological perturbation theory that is applicable to subhorizon perturbations, gradient expansion is often used to tackle superhorizon perturbations. In this paper, we have developed the superhorizon gradient expansion formalism for Galileon inflation, a novel inflation model characterized by its higher order derivative interactions. This model is inspired by a new class of infrared modifications of gravity, called (generalized) Galileon models, introduced to explain the late time accelerated cosmic expansion. There are many interesting features in Galilleon inflation, including new shapes of non-Gaussianty [45]. We have solved the equations of motion of Galileon infl ation up to se cond order in gradient expansion in the synchronous gauge, and obtained the general solution without imposing extra conditions on the first order quantities. We have identified the physical degrees of freedom in the solution, taking particular care in keeping track of the residual gauge freedom left after imposing the synchronous gauge condition. We have also defined a curvature perturbation $\mathcal{R}^{(1)}$ conserved up to first order. With the formalism developed here, one can evaluate non-Gassianities in Galileon inflation at superhorizon scales, and, combined with the conventional non-linear perturbation analysis, one can then use the existing data to constrain the model parameters.

A typical way to generate non-Gaussianity is to have multiple fields. Non-Gaussianity in the multi-Galileon model has been discussed [46]. It would be interesting to develop a superhorizon gradient expansion for multi-Galileon inflation. Another key feature of Galileon

gravity is that it is supposed to give rise to $\mathcal{O}(1)$ corrections to general relativity at large distances and yet satisfies stringent constraints at short distances, such as in the solar system where any modification is typically constrained below $\mathcal{O}(10^{-5})$. This is achieved due to the high degree of non-linearity of the Galileon derivative interactions and the phenomenon is called the Vainshtein mechanism, originally discovered in massive gravity [43, 55]. This mechanism is not easy to see in perturbation theory due to its non-linear nature, and the full non-linear problem is difficult to solve. It would be interesting to use the gradient expansion in order to get a deeper understanding of the behaviour of these non-linear interactions, at least in the regime where it is applicable.

A Dependence of equations of motion in general covariant scalar-tensor theory

Consider a general covariant scalar-tensor theory of ϕ and $g_{\mu\nu}$, given by the action $S(\phi, g_{\mu\nu})$. The equations of motion for this system are

$$\mathcal{E} = \frac{1}{\sqrt{-g}}\frac{\delta S}{\delta \phi} = 0, \qquad \mathcal{E}_{\mu\nu} = \frac{1}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} = 0, \tag{A.1}$$

with the variation of the action (modulo boundary terms) is given by

$$\delta S = \int \mathrm{d}x^D \sqrt{-g}\left(\mathcal{E}\delta\phi + \mathcal{E}_{\mu\nu}\delta g^{\mu\nu}\right). \tag{A.2}$$

Now, we assume this action is covariant, which means it is invariant under the following transformation ($\delta_\xi x^\mu = -\xi^\mu$)

$$\delta_\xi \phi = \mathcal{L}_\xi \phi = \xi^\mu \nabla_\mu \phi, \qquad \delta_\xi g^{\mu\nu} = \mathcal{L}_\xi g^{\mu\nu} = 2\nabla^{(\mu}\xi^{\nu)}. \tag{A.3}$$

That is, we have

$$\delta_\xi S = \int \mathrm{d}^D x \sqrt{-g}\left(\mathcal{E}\cdot \xi^\nu \nabla_\nu \phi + \mathcal{E}_{\mu\nu}\cdot 2\nabla^\mu \xi^\nu\right) = 0. \tag{A.4}$$

After integration by parts, we get $\int \mathrm{d}^D x\sqrt{-g}\xi^\nu\left(\mathcal{E}\nabla_\nu\phi + 2\nabla^\mu \mathcal{E}_{\mu\nu}\right) = 0$. Since ξ^ν is arbitrary, we have

$$\mathcal{E}\nabla_\nu \phi = 2\nabla^\mu \mathcal{E}_{\mu\nu}. \tag{A.5}$$

So if the Einstein equations are satisfied ($\mathcal{E}_{\mu\nu} = 0$), the scalar equation of motion is automatically satisfied ($\mathcal{E} = 0$).

B Gradient expansion of the equations of motion in synchronous gauge

Here we list the Einstein tensor, the effective energy momentum tensor and the scalar current (the t component) up to order $\mathcal{O}(\epsilon^2)$ in the superhorizon gradient expansion. As in the main

text, we suppress the background quantities' order indication $^{(0)}$. For example, $\dot{\phi}^{(0)}$ *is written as* $\dot{\phi}$

The quantities at the $\mathcal{O}(\epsilon^0)$ order:

$$G^{(0)}_{tt} = 3H^2, \tag{B.1}$$

$$G^{(0)}_{ti} = 0, \tag{B.2}$$

$$G^{(0)i}{}_{j} = -\delta^i_j \left(2\dot{H} + 3H^2\right), \tag{B.3}$$

$$T^{(0)}_{\phi}{}_{tt} = -K + 2K_X X + 6H\dot{\phi}G_X X, \tag{B.4}$$

$$T^{(0)}_{\phi}{}_{ti} = 0, \tag{B.5}$$

$$T^{(0)i}_{\phi}{}_{j} = K - 2G_X X\ddot{\phi}, \tag{B.6}$$

$$J^{t(0)} = K_X\dot{\phi} + 6HG_X X. \tag{B.7}$$

The quantities at the $\mathcal{O}(\epsilon)$ order:

$$G^{(1)}_{tt} = 6H\dot{\zeta}^{(1)}, \tag{B.8}$$

$$G^{(1)}_{ti} = \frac{2}{3}\mathrm{D}_i\mathcal{K}^{(0)} = 0, \tag{B.9}$$

$$G^{(1)i}{}_{j} = -2\left(\ddot{\zeta}^{(1)} + 3H\dot{\zeta}^{(1)}\right)\delta^i_j - \left(3HA^{(1)i}{}_j + \dot{A}^{(1)i}{}_j\right), \tag{B.10}$$

$$T^{(1)}_{\phi}{}_{tt} = \left(\dot{\phi}K_X + 2\dot{\phi}K_{XX}X + 18HG_X X + 12HG_{XX}X^2\right)\dot{\phi}^{(1)} + 6\dot{\phi}G_X X\dot{\zeta}^{(1)}, \tag{B.11}$$

$$T^{(1)}_{\phi}{}_{ti} = 0, \tag{B.12}$$

$$T^{(1)i}_{\phi}{}_{j} = \left[\left(\dot{\phi}K_X - 2\dot{X}(G_X + G_{XX}X)\right)\dot{\phi}^{(1)} - 2G_X X\ddot{\phi}^{(1)}\right]\delta^i_j, \tag{B.13}$$

$$J^{t(1)} = \left(K_X + 2K_{XX}X + 6H\dot{\phi}(G_X + G_{XX}X)\right)\dot{\phi}^{(1)} + 6G_X X\dot{\zeta}^{(1)}. \tag{B.14}$$

The quantities at the $\mathcal{O}(\epsilon^2)$ order:

$$G^{(2)}_{tt} = \frac{1}{2}\left({}^{[3]}R^{(2)} + 6\left(\dot{\zeta}^{(1)}\right)^2 + 12H\dot{\zeta}^{(2)} - A^{(1)k}{}_l A^{(1)l}{}_k\right), \tag{B.15}$$

$$G^{(2)}_{ti} = -2\mathrm{D}_i\dot{\zeta}^{(1)} - \mathrm{D}_k A^{(1)k}{}_i, \tag{B.16}$$

$$\begin{aligned} G^{(2)i}{}_{j} &= {}^{[3]}G^{(2)i}{}_j - \dot{A}^{(2)i}{}_j - 3HA^{(2)i}{}_j - 3\dot{\zeta}^{(1)}A^{(1)i}{}_j \\ &\quad - \left(2\ddot{\zeta}^{(2)} + 6H\dot{\zeta}^{(2)} + 3\left(\dot{\zeta}^{(1)}\right)^2 + \frac{1}{2}A^{(1)k}{}_l A^{(1)l}{}_k\right)\delta^i_j, \end{aligned} \tag{B.17}$$

$$\begin{aligned} T^{(2)}_{\phi}{}_{tt} &= (\dot{\phi}K_X + 2\dot{\phi}K_{XX}X + 18HG_X X + 12HG_{XX}X^2)\dot{\phi}^{(2)} + 6\dot{\phi}G_X X\dot{\zeta}^{(2)} \\ &\quad + \left(\frac{1}{2}K_X + 4K_{XX}X + 2K_{XXX}X^2 + 9H\dot{\phi}G_X + 21H\dot{\phi}G_{XX}X + 6H\dot{\phi}G_{XXX}X^2\right)\left(\dot{\phi}^{(1)}\right)^2 \\ &\quad + (18G_X X + 12G_{XX}X^2)\dot{\zeta}^{(1)}\dot{\phi}^{(1)}, \end{aligned} \tag{B.18}$$

$$T^{(2)}_{\phi}{}_{ti} = (\dot{\phi}K_X + 6HG_X X)\partial_i\phi^{(1)} - 2G_X X\partial_i\dot{\phi}^{(1)}, \tag{B.19}$$

$$\begin{aligned} T^{(2)i}_{\phi}{}_{j} &= \delta^i_j\Big[-2G_X X\ddot{\phi}^{(2)} + \left(\dot{\phi}K_X - 2\dot{X}(G_X + G_{XX}X)\right)\dot{\phi}^{(2)} - 2\dot{\phi}(G_X + G_{XX}X)\dot{\phi}^{(1)}\ddot{\phi}^{(1)} \\ &\quad + \left(\frac{1}{2}K_X + K_{XX}X - \ddot{\phi}G_X - 5\ddot{\phi}G_{XX}X - 2\ddot{\phi}G_{XXX}X^2\right)\left(\dot{\phi}^{(1)}\right)^2\Big], \end{aligned} \tag{B.20}$$

$$
\begin{aligned}
J^{t(2)} &= 6G_X X\dot{\zeta}^{(2)} + \left(K_X + 2K_{XX}X + 6H\dot{\phi}G_X + 6H\dot{\phi}G_{XX}X\right)\dot{\phi}^{(2)} \\
&+ \left(\frac{3}{2}\dot{\phi}K_{XX} + \dot{\phi}K_{XXX}X + 3HG_X + 15HG_{XX}X + 6HG_{XXX}X^2\right)\left(\dot{\phi}^{(1)}\right)^2 \\
&+ \left(6\dot{\phi}G_X + 6\dot{\phi}G_{XX}X\right)\dot{\zeta}^{(1)}\dot{\phi}^{(1)}. \qquad \text{(B.21)}
\end{aligned}
$$

Acknowledgments

We would like to thank Tsutomu Kobayashi, Arif Mohd, Shinji Mukohyama, Paul Saffin and Alessanda Silvestri for helpful discussions. The research leading to these results has received funding from the European Research Council under the European Union's Seventh Framework Programme (FP7/2007-2013) / ERC grant agreement nř 306425 "Challenging General Relativity" and from the Marie Curie Career Integration Grant LIMITSOFGR-2011-TPS.

References

[1] J. M. Maldacena, *Non-Gaussian features of primordial fluctuations in single field inflationary models, JHEP* **0305**, 013 (2003) [astro-ph/0210603].

[2] X. Chen, R. Easther and E. A. Lim, *Large Non-Gaussianities in Single Field Inflation, JCAP* **0706**, 023 (2007) [astro-ph/0611645].

[3] C. L. Bennett, D. Larson, J. L. Weiland, N. Jarosik, G. Hinshaw, N. Odegard, K. M. Smith and R. S. Hill *et al.*, *Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Final Maps and Results*, arXiv:1212.5225 [astro-ph.CO].

[4] P. A. R. Ade *et al.* [Planck Collaboration], arXiv:1303.5084 [astro-ph.CO].

[5] N. Bartolo, E. Komatsu, S. Matarrese and A. Riotto, *Non-Gaussianity from inflation: Theory and observations, Phys. Rept.* **402**, 103 (2004) [astro-ph/0406398].

[6] X. Chen, *Primordial Non-Gaussianities from Inflation Models, Adv. Astron.* **2010**, 638979 (2010) [arXiv:1002.1416 [astro-ph.CO]].

[7] V. Acquaviva, N. Bartolo, S. Matarrese and A. Riotto, *Second order cosmological perturbations from inflation, Nucl. Phys. B* **667**, 119 (2003) [astro-ph/0209156].

[8] K. A. Malik and D. Wands, *Evolution of second-order cosmological perturbations, Class. Quant. Grav.* **21**, L65 (2004) [astro-ph/0307055].

[9] K. Nakamura, *Second-order gauge invariant cosmological perturbation theory: Einstein equations in terms of gauge invariant variables, Prog. Theor. Phys.* **117**, 17 (2007) [gr-qc/0605108].

[10] K. A. Malik, *Gauge-invariant perturbations at second order: Multiple scalar fields on large scales, JCAP* **0511**, 005 (2005) [astro-ph/0506532].

[11] D. Langlois and F. Vernizzi, *Nonlinear perturbations of cosmological scalar fields," JCAP* **0702**, 017 (2007) [astro-ph/0610064].

[12] A. J. Christopherson, K. A. Malik, D. R. Matravers and K. Nakamura, *Comparing different formulations of non-linear cosmological perturbation theory, Class. Quant. Grav.* **28**, 225024 (2011) [arXiv:1101.3525 [astro-ph.CO]].

[13] E. M. Lifshitz and I. M. Khalatnikov, *Investigations in relativistic cosmology, Adv. Phys.* **12**, 185 (1963).

[14] K. Tomita, *Evolution of Irregularities in a Chaotic Early Universe, Prog. Theor. Phys.* **54**, 730 (1975).

[15] D. S. Salopek and J. R. Bond, *Nonlinear evolution of long wavelength metric fluctuations in inflationary models, Phys. Rev. D* **42**, 3936 (1990).

[16] G. L. Comer, N. Deruelle, D. Langlois and J. Parry, *Growth or decay of cosmological inhomogeneities as a function of their equation of state, Phys. Rev. D* **49**, 2759 (1994).

[17] N. Deruelle and D. Langlois, *Long wavelength iteration of Einstein's equations near a space-time singularity, Phys. Rev. D* **52**, 2007 (1995) [gr-qc/9411040].

[18] D. H. Lyth, K. A. Malik and M. Sasaki, *A General proof of the conservation of the curvature perturbation, JCAP* **0505**, 004 (2005) [astro-ph/0411220].

[19] O. Iguchi, H. Ishihara and J. Soda, *Inhomogeneity of spatial curvature for inflation, Phys. Rev. D* **55**, 3337 (1997) [gr-qc/9606012].

[20] Y. Tanaka and M. Sasaki, *Gradient expansion approach to nonlinear superhorizon perturbations, Prog. Theor. Phys.* **117**, 633 (2007) [gr-qc/0612191].

[21] Y. -i. Takamizu and S. Mukohyama, *Nonlinear superhorizon perturbations of non-canonical scalar field, JCAP* **0901**, 013 (2009) [arXiv:0810.0746 [gr-qc]].

[22] K. Izumi and S. Mukohyama, *Nonlinear superhorizon perturbations in Horava-Lifshitz gravity, Phys. Rev. D* **84**, 064025 (2011) [arXiv:1105.0246 [hep-th]].

[23] A. E. Gumrukcuoglu, S. Mukohyama and A. Wang, *General relativity limit of Horava-Lifshitz gravity with a scalar field in gradient expansion, Phys. Rev. D* **85**, 064042 (2012) [arXiv:1109.2609 [hep-th]].

[24] Y. -i. Takamizu and T. Kobayashi, *Nonlinear superhorizon curvature perturbation in generic single-field inflation*, arXiv:1301.2370 [gr-qc].

[25] A. Naruko, Y. -i. Takamizu and M. Sasaki, *Beyond δ N formalism*, arXiv:1210.6525 [astro-ph.CO].

[26] D. Wands, K. A. Malik, D. H. Lyth and A. R. Liddle, *A New approach to the evolution of cosmological perturbations on large scales, Phys. Rev. D* **62**, 043527 (2000) [astro-ph/0003278].

[27] A. A. Starobinsky, *Multicomponent de Sitter (Inflationary) Stages and the Generation of Perturbations, JETP Lett.* **42**, 152 (1985) [Pisma Zh. Eksp. Teor. Fiz. **42**, 124 (1985)].

[28] M. Sasaki and E. D. Stewart, *A General analytic formula for the spectral index of the density perturbations produced during inflation, Prog. Theor. Phys.* **95**, 71 (1996) [astro-ph/9507001].

[29] S. M. Leach, M. Sasaki, D. Wands and A. R. Liddle, *Enhancement of superhorizon scale inflationary curvature perturbations, Phys. Rev. D* **64**, 023512 (2001) [astro-ph/0101406].

[30] A. Nicolis, R. Rattazzi and E. Trincherini, *The Galileon as a local modification of gravity, Phys. Rev. D* **79**, 064036 (2009) [arXiv:0811.2197 [hep-th]].

[31] C. Deffayet, G. Esposito-Farese and A. Vikman, *Covariant Galileon, Phys. Rev. D* **79**, 084003 (2009) [arXiv:0901.1314 [hep-th]].

[32] T. Kobayashi, M. Yamaguchi and J. 'i. Yokoyama, *G-inflation: Inflation driven by the Galileon field, Phys. Rev. Lett.* **105**, 231302 (2010) [arXiv:1008.0603 [hep-th]].

[33] C. Deffayet, S. Deser and G. Esposito-Farese, *Generalized Galileons: All scalar models whose curved background extensions maintain second-order field equations and stress-tensors, Phys. Rev. D* **80**, 064015 (2009) [arXiv:0906.1967 [gr-qc]].

[34] C. Deffayet, X. Gao, D. A. Steer and G. Zahariade, *From k-essence to generalised Galileons, Phys. Rev. D* **84**, 064039 (2011) [arXiv:1103.3260 [hep-th]].

[35] T. Kobayashi, M. Yamaguchi and J. 'i. Yokoyama, *Generalized G-inflation: Inflation with the most general second-order field equations, Prog. Theor. Phys.* **126**, 511 (2011) [arXiv:1105.5723 [hep-th]].

[36] G. W. Horndeski, *Second-order scalar-tensor field equations in a four-dimensional space, Int. J. Theor. Phys.* **10**, 363 (1974).

[37] C. de Rham and A. J. Tolley, *DBI and the Galileon reunited, JCAP* **1005**, 015 (2010) [arXiv:1003.5917 [hep-th]].

[38] A. Padilla, P. M. Saffin and S. -Y. Zhou, *Bi-galileon theory II: Phenomenology, JHEP* **1101**, 099 (2011) [arXiv:1008.3312 [hep-th]].

[39] C. Deffayet, S. Deser and G. Esposito-Farese, *Arbitrary p-form Galileons, Phys. Rev. D* **82**, 061501 (2010) [arXiv:1007.5278 [gr-qc]].

[40] A. Padilla, P. M. Saffin and S. -Y. Zhou, *Multi-galileons, solitons and Derrick's theorem, Phys. Rev. D* **83**, 045009 (2011) [arXiv:1008.0745 [hep-th]].

[41] S. -Y. Zhou and E. J. Copeland, *Galileons with Gauge Symmetries, Phys. Rev. D* **85**, 065002 (2012) [arXiv:1112.0968 [hep-th]].

[42] G. Goon, K. Hinterbichler, A. Joyce and M. Trodden, *Galileons as Wess-Zumino Terms, JHEP* **1206**, 004 (2012) [arXiv:1203.3191 [hep-th]].

[43] K. Hinterbichler, *Theoretical Aspects of Massive Gravity, Rev. Mod. Phys.* **84**, 671 (2012) [arXiv:1105.3735 [hep-th]].

[44] P. Creminelli, A. Nicolis and E. Trincherini, *Galilean Genesis: An Alternative to inflation, JCAP* **1011**, 021 (2010) [arXiv:1007.0027 [hep-th]].

[45] P. Creminelli, G. D'Amico, M. Musso, J. Norena and E. Trincherini, *Galilean symmetry in the effective theory of inflation: new shapes of non-Gaussianity, JCAP* **1102**, 006 (2011) [arXiv:1011.3004 [hep-th]].

[46] S. Mizuno and K. Koyama, *Primordial non-Gaussianity from the DBI Galileons, Phys. Rev. D* **82**, 103518 (2010) [arXiv:1009.0677 [hep-th]].

[47] K. Kamada, T. Kobayashi, M. Yamaguchi and J. 'i. Yokoyama, *Higgs G-inflation, Phys. Rev. D* **83**, 083515 (2011) [arXiv:1012.4238 [astro-ph.CO]].

[48] A. De Felice and S. Tsujikawa, *Shapes of primordial non-Gaussianities in the Horndeski's most general scalar-tensor theories*, arXiv:1301.5721 [hep-th].

[49] T. Kobayashi, M. Yamaguchi and J. 'i. Yokoyama, *rimordial non-Gaussianity from G-inflation, Phys. Rev. D* **83**, 103524 (2011) [arXiv:1103.1740 [hep-th]].

[50] X. Gao and D. A. Steer, *Inflation and primordial non-Gaussianities of 'generalized Galileons', JCAP* **1112**, 019 (2011) [arXiv:1107.2642 [astro-ph.CO]].

[51] C. Armendariz-Picon, T. Damour and V. F. Mukhanov, *k - inflation, Phys. Lett. B* **458**, 209 (1999) [hep-th/9904075].

[52] M. A. Luty, M. Porrati and R. Rattazzi, *Strong interactions and stability in the DGP model, JHEP* **0309**, 029 (2003) [hep-th/0303116].

[53] E. Gourgoulhon, *3+1 formalism and bases of numerical relativity*, gr-qc/0703035 [GR-QC].

[54] G. I. Rigopoulos and E. P. S. Shellard, *The separate universe approach and the evolution of nonlinear superhorizon cosmological perturbations, Phys. Rev. D* **68**, 123518 (2003) [astro-ph/0306620].

[55] A. I. Vainshtein, *To the problem of nonvanishing gravitation mass, Phys. Lett. B* **39**, 393 (1972).